**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans: - we have a normal distribution is mean = 45 and variance = 8. Let x be the amount of time is takes to complete the repair on customer’s car. To finish in one hour you must have x<=50 so the question is to find p(x>50).**

**P(x>50) =1-p(x<=50)**

**Z=(x-45)/0.8**

**P(x<=50) = p(z<=50-45)/8) = 73.4%**

**Probability that the service manager will not meet his demand will be = 100-73.4 = 0.2676.**

**B) 0.2676**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans: - Both statement are true, let’s take a briefly explain**

**we have a normal distribution with mean=38 and variance = 6. Let x be the number of employees. So according to question**

**a)Probability of employee greater then age of 44= P(x>44)**

**p(x>44) = 1-P(x<=44).**

**Z = (x-38)/6**

**P (x<=44)= P(z<=(44-38))/6 = P(z<= 1) = 84.1345%**

**So the probability of number of employees between 38-44 years of age = p(x>44)-0.5 = 84.1345-0.5 = 34.1245%**

**Therefore the statement that “More employees at the processing center are older than 44 than between 38 and 44” is TRUE.**

**b)probability of employees less than age of 30 = p(x<30)**

**z = (x=38)/6**

**P(x<=30) = P(x<=30-38)/6) = p(z <= -1.333) = 9.12%**

**So the number of employees with probability 0.912 of then being under age 30 = 0.912\*400 = 36.48.**

**Therefore the statement B of the question is also TRUE.**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans: - Given that *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) assume that X1 and X2**

**Have same μ=1 and *σ*** =2

**Var(2\*X1)=2^2\*Var(1,2^2)**

**Var(4,16)**

**Var(X1+X2)= Var(X1)+Var(X2)**

**= Var(1,4)+Var(1,4)**

**=Var(2,8)**

**It means that var 2X1 has to 2 times more variance value than the Var(X1+X2)**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans: - Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between then is 0.99 we have to work out in revers order.**

**The probability of getting value between a and b should be 0.99**

**So the Probability of going wrong, or the probability outside the a and b area is 0.01**

**The probability towards left from a= -0.005**

**The probability towards left from b= +0.005**

**So since we have probability of a and b, we need to calculate x, the random variable at a and b which has got these probabilities.**

**By finding the standard normal variable Z(Z value), we can calculate the x value.**

**Z=(X-** **μ)/*****σ***

**For probabilities 0.005 the Z value is -2.57**

**Z\**σ +* μ=x**

**Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4**

**Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6**

**So option D is correct.**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans: Attached Q5.Set-2 Assignment file IPYNB**